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§ 2.3.5 Summary : Electrostatic Boundary Conditions

18 century , Gray

⇒ a. conductor ➡ surface  ← charge 只存在表面

b. Dielectric ➡ volume  ← charge 分佈體積內

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Boundary condition.

Summary of 2.1 - 2.3

(1) $\vec{E} = -\vec{\nabla} U$; $\vec{E} \longleftrightarrow U$

(2) $V = \frac{1}{4\pi\epsilon_0}\int \frac{\rho}{r} dv'$; $U \longleftrightarrow \rho$

(3) $\vec{E} = \frac{1}{4\pi\epsilon_0}\int \frac{\rho \hat{r}}{r^2} dv'$; $\vec{E} \longleftrightarrow \rho$

→ Example

$U = -\int \vec{E}\cdot d\vec{l}$

$\frac{\rho}{\epsilon_0} = \vec{\nabla}\cdot\vec{E}$

$\frac{\rho}{\epsilon_0} = \vec{\nabla}^2 U$

Gradient : $f(b) - f(a) = \int_a^b (\nabla f)\cdot d\vec{l}$

Divergency : $\vec{\nabla}\cdot\vec{A} = \frac{\Phi = A\cdot d\vec{a}}{volume}$, $\int \vec{\nabla}\cdot\vec{A}\, dv = \oint \vec{A}\cdot d\vec{a}$

Curl : $\nabla\times A = \frac{\vec{A}\cdot d\vec{l}}{a}$, $\int \nabla\times\vec{A}\, da = \oint \vec{A}\, d\vec{l}$

§ 2.3.5 Boundary condition

內部電場為零 ➡ 靜電學 ➡ 邊界條件

States that two cases

1. $\int \vec{E}\cdot d\vec{a}$, Divergency

* If a conductor :
inside the box contribute nothing to the flux as the limit $d \to 0$

$$\oint \vec{E}_n \cdot d\vec{a} = \frac{Q_{enc}}{\epsilon_0}$$

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where E_n is perpendicular to the surface
we can make a conclusion :
the normal component of $\vec{E}_n$ is "discontinous".

2. $\oint \vec{E}_t \cdot dl$ ← Curl

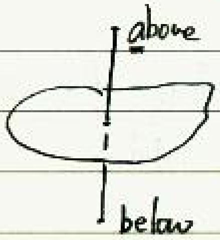
$\oint \vec{E}_t \cdot d\vec{l} = 0$

plane $\oint$ $\oint$

* E-field is zero at tangential components $\vec{E}_t$
(平面)

And the tangential component of $\vec{E}$ is always "continous". so $\vec{E}_{at} = \vec{E}_{bt}$

3. Boundary condition (B.C. 條件)
The B.C. on $\vec{E}$ can be combined into a single formula.



$\vec{E}_{at} - \vec{E}_{bt} = 0$
$\vec{E}_{an} - \vec{E}_{bn} = \sigma/\epsilon_0$

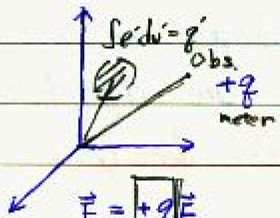
$(\vec{E}_{at} + \vec{E}_{an}) - (\vec{E}_{bn} + \vec{E}_{bt}) = \frac{\sigma}{\epsilon_0}$

$\Rightarrow V_a - V_b = -\int_b^a \vec{E} \cdot d\vec{l}$, $\boxed{\vec{E}_n = -\vec{\nabla}_n V}$

$(-\nabla_n V_a) - (-\nabla_n V_b) = -\frac{\sigma}{\epsilon_0}\hat{n}$

$\frac{\partial V_a}{\partial n} - \frac{\partial V_b}{\partial n} = -\frac{\sigma}{\epsilon_0}\hat{n}$. $\frac{\partial V}{\partial n} = \nabla V \cdot \hat{n}$

§ 2.4 Work and Energy in Electrostatics (靜電)



$\vec{F} = \boxed{+q}\vec{E}$

$\boxed{\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{q_1}{r^2}}$

* The electric potential is the potential energy (q') per unit charge (+q) and the potential difference between +q and q' in an electric field is equal to the work per unit charge.

$$\underset{+q}{V(\vec{r}_2)} - \underset{q'}{V(\vec{r}_1)} = \frac{W}{+q}$$

or the work can be represented (-Q) as

$$W = \int \vec{F}\cdot d\vec{l} = \int -Q\vec{E}\cdot d\vec{l} = Q\,[V(b) - V(a)]$$

$\boxed{\text{using gradient rule}}$

$$\frac{W}{Q} = V(b) - V(a)$$

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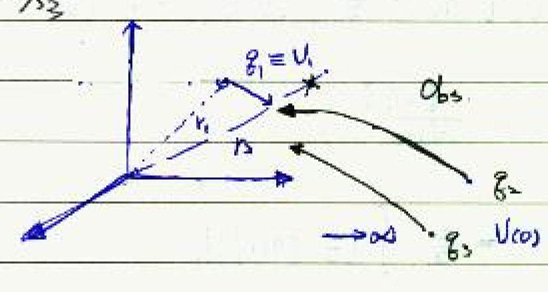

$$W_{12} = q_2 \cdot V(r_{12}) = q_2 \cdot \frac{1}{4\pi\varepsilon_0}\cdot\frac{q_1}{r_{12}}$$

$$W_{123} = q_3\,(V_{13} + V_{23}) = q_3\,\frac{1}{4\pi\varepsilon_0}\left[\frac{q_1}{r_{13}} + \frac{q_2}{r_{23}}\right)$$

The total work for multi-charges can be described as

$$W = \frac{q_2}{4\pi\varepsilon_0}\frac{q_1}{r_{12}} + \frac{q_3}{4\pi\varepsilon_0}\left[\frac{q_1}{r_{13}} + \frac{q_2}{r_{23}}\right] + \cdots = \frac{1}{4\pi\varepsilon_0}\sum_i q_i \left[\sum_{j<i}\frac{q_j}{r_{ij}}\right]$$

$$= \frac{1}{4\pi\varepsilon_0}\left(\frac{1}{2}\right)\sum_i\sum_j \frac{q_i q_j}{r_{ij}} = \frac{1}{2}\sum q_i V(r_j)$$

有了人没点到 ?

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* The factor $(\frac{1}{2})$ appears in the expression because each pair is accounted twice.

The equation of (2.43) if the charges are not localized as point charges but are distributed with volume density ρ and surface density σ.

(2.43) ⟶ rewritten as.

$$W = \frac{1}{2}\int \rho \, dv \cdot V + \frac{1}{2}\int \sigma \, ds \cdot V$$

From Gauss' law

$$\frac{1}{2}\int \rho \, dv \; V = \frac{1}{2}\epsilon_0 \int (\vec{\nabla}\cdot\vec{E}) \, dv \cdot V$$

Note $\nabla\cdot(V\vec{E}) = V(\nabla\cdot\vec{E}) + \vec{E}\cdot(\nabla V)$

$(\nabla\cdot\vec{E})V = \vec{\nabla}\cdot(V\vec{E}) = \vec{E}\cdot(\nabla V)$

$$\frac{1}{2}\int \epsilon_0 (\vec{\nabla}\cdot\vec{E}) V \, dv = \frac{\epsilon_0}{2}\int [\vec{\nabla}\cdot(\vec{E}V) - \vec{E}\cdot(\nabla V)] \, dv$$

(Divergency $\vec{\nabla}\cdot\vec{A} = \frac{\Phi = \vec{A}\cdot\vec{S}}{volume}$)

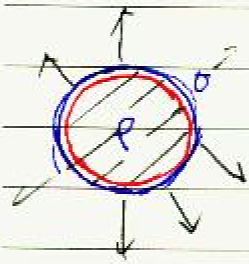
$$= \frac{\epsilon_0}{2}\int V\vec{E} \, d\vec{s} - \frac{\epsilon_0}{2}\int [\vec{E}\cdot\overset{-\vec{E}}{(\nabla V)}] \, dv$$

Total work is

$$W = \frac{\epsilon_0}{2}\int V\vec{E}\cdot d\vec{s} + \frac{\epsilon_0}{2}\int \vec{E}\cdot\vec{E} \, dv + \frac{1}{2}\int \sigma V \, ds$$

If in B.C., E_n is the normal component of $\vec{E}$ out of volume V

$$E_n = \frac{\sigma}{\epsilon_0}, \quad W = -\frac{1}{2}\int V\sigma\, d\vec{s} + \frac{\epsilon}{2}\int \vec{E}\cdot\vec{E}\, dv' + \frac{1}{2}\int \sigma V\, ds$$

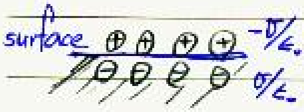


Hence the work

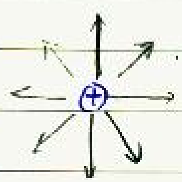
$$W \equiv \frac{\epsilon}{2}\int \vec{E}\cdot\vec{E}\, dv' = \frac{\epsilon}{2}\int E^2\, dv' = \int u\, dv'$$

u : energy density $= \frac{1}{2}\epsilon_0 E^2$

$W = mc^2 \Rightarrow W/mc^2 =$ work / charge



We can not apply $W = \frac{\epsilon}{2}\int E^2\, dv'$ to a point charge for it shows that energy of a point charge is



$$W = \frac{\epsilon}{2}\int_0^{r}\int_0^{\frac{\pi}{2}}\int_0^{a}\left(\frac{q}{4\pi\epsilon_0 r^2}\right)^2 r^2\, dr \sin\theta\, d\theta\, d\phi$$

$$= \frac{q^2}{8\pi\epsilon_0}\int_0^{\infty}\frac{1}{r^2}\, dr = \infty$$